TÜBİTAK
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BLOKZİNCİR
Araştırma Laboratuvarı

Zero-Knowledge Proofs in Blockchains (Blokzincirlerde Sıfır Bilgi İspatları)

2. Uusal Blokzincir Çalıştayı, İstanbul, 2019
26 Eylül 2019

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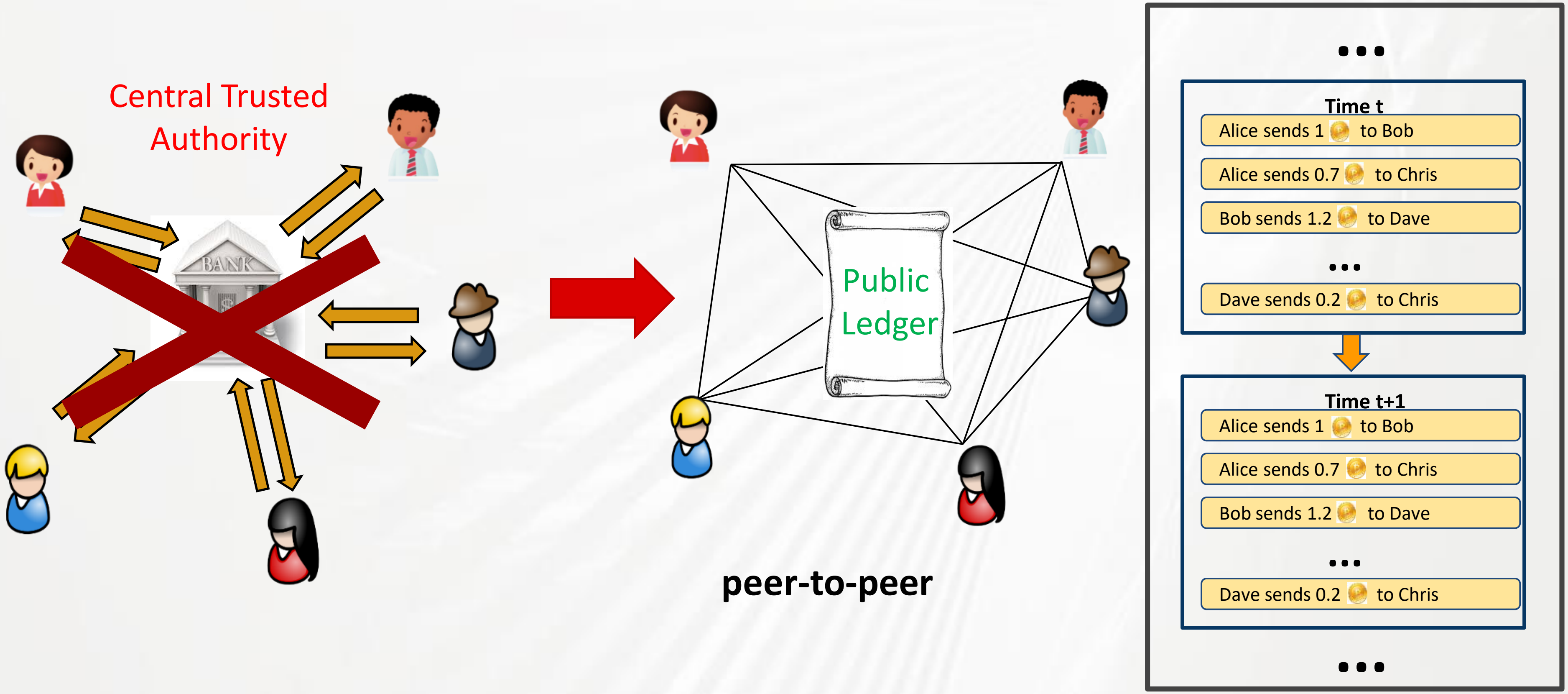
Privacy against who?

❑ Decentralized, no trusted server



…

Time t

Alice sends 1 ● to Bob

Alice sends 0.7 ● to Chris

Bob sends 1.2 ● to Dave

…

Dave sends 0.2 ● to Chris

Time t+1

Alice sends 1 ● to Bob

Alice sends 0.7 ● to Chris

Bob sends 1.2 ● to Dave

…

Dave sends 0.2 ● to Chris

…

Centralized: Reveal amount, sender/receiver info to the bank

De-centralized: Reveal amount, sender/receiver info to everyone

Transaction amounts available in the clear

Everyone can see the payer, payee, and value

Business implications:

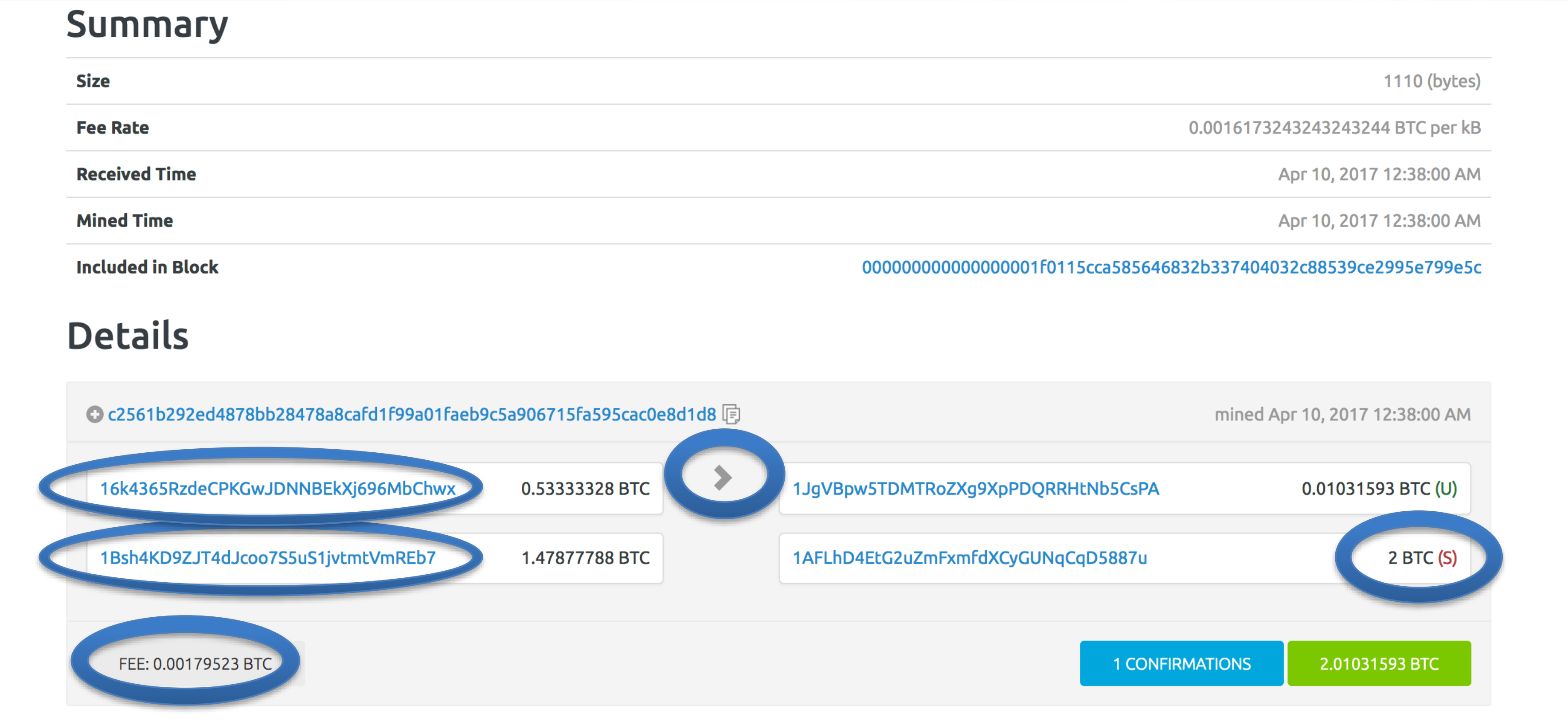
- Company pays employees in Bitcoin.
  ⇒ all salaries are public

- Public supply chain prices:
  - How much does Ford pay its supplier for tires?

Problem: Every transaction ever made is **recorded forever**

Bitcoin is neither confidential nor anonymous

Anonymity vs Pseudnymity



Summary

Size	1110 (bytes)
Fee Rate	0.0016173243243243244 BTC per kB
Received Time	Apr 10, 2017 12:38:00 AM
Mined Time	Apr 10, 2017 12:38:00 AM
Included in Block	000000000000000001f0115cca585646832b337404032c88539ce2995e799e5c

Details

c2561b292ed4878bb28478a8cafd1f99a01faeb9c5a906715fa595cac0e8d1d8 — mined Apr 10, 2017 12:38:00 AM

16k4365RzdeCPKGwJDNNBEkXj696MbChwx	0.53333328 BTC	>	1JgVBpw5TDMTRoZXg9XpPDQRRHtNb5CsPA	0.01031593 BTC (U)
1Bsh4KD9ZJT4dJcoo7S5uS1jvtmtVmREb7	1.47877788 BTC		1AFLhD4EtG2uZmFxmfdXCyGUNqCqD5887u	2 BTC (S)

FEE: 0.00179523 BTC

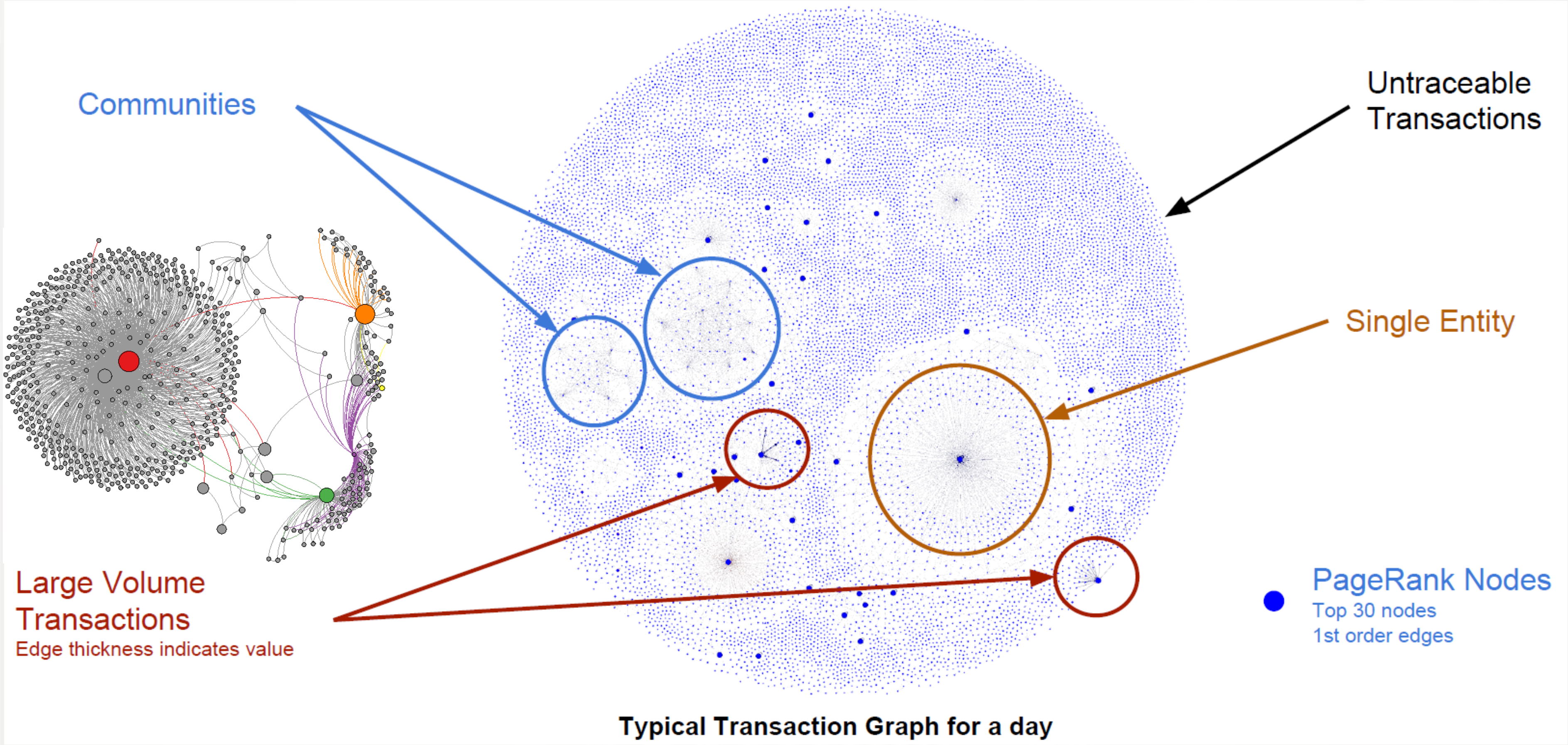
1 CONFIRMATIONS

2.01031593 BTC

Bitcoin only offers pseudo-anonymity. Transactions are linkable and can be potentially de-anonymized

Transaction Flow Graph [Maxwell 2016]

Pseudonymity cannot provide Anonymity!!

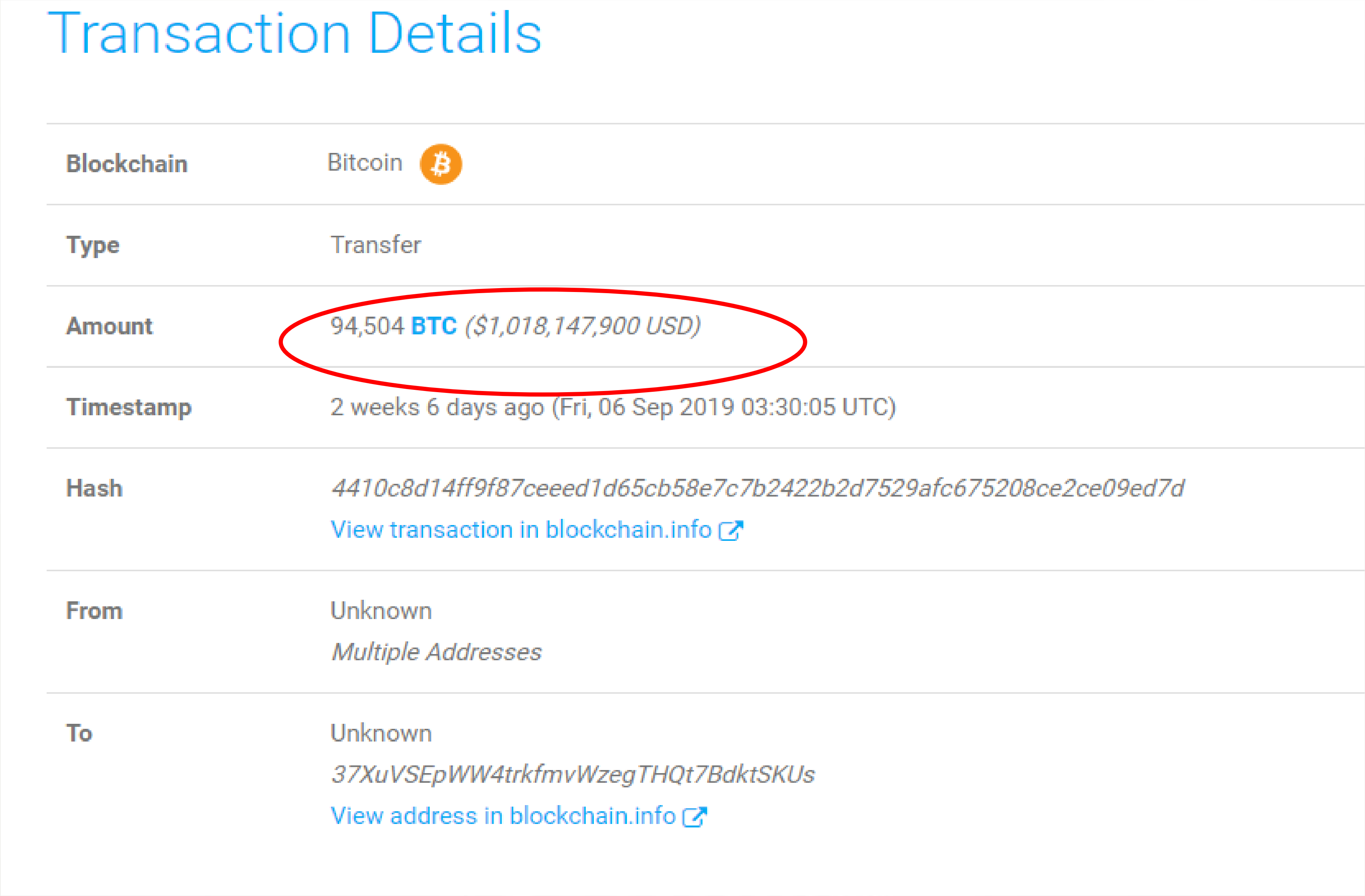


Typical Transaction Graph for a day

- **Transaction graph is still public**

[Reid Martin 11] [Barber Boyen Shi Uzun 12] [Ron Shamir 12] [Ron Shamir 13] [Meiklejohn Pomarole Jordan Levchenko McCoy Voelker Savage 13] [Ron Shamir 14]

Breaking News on Transactions

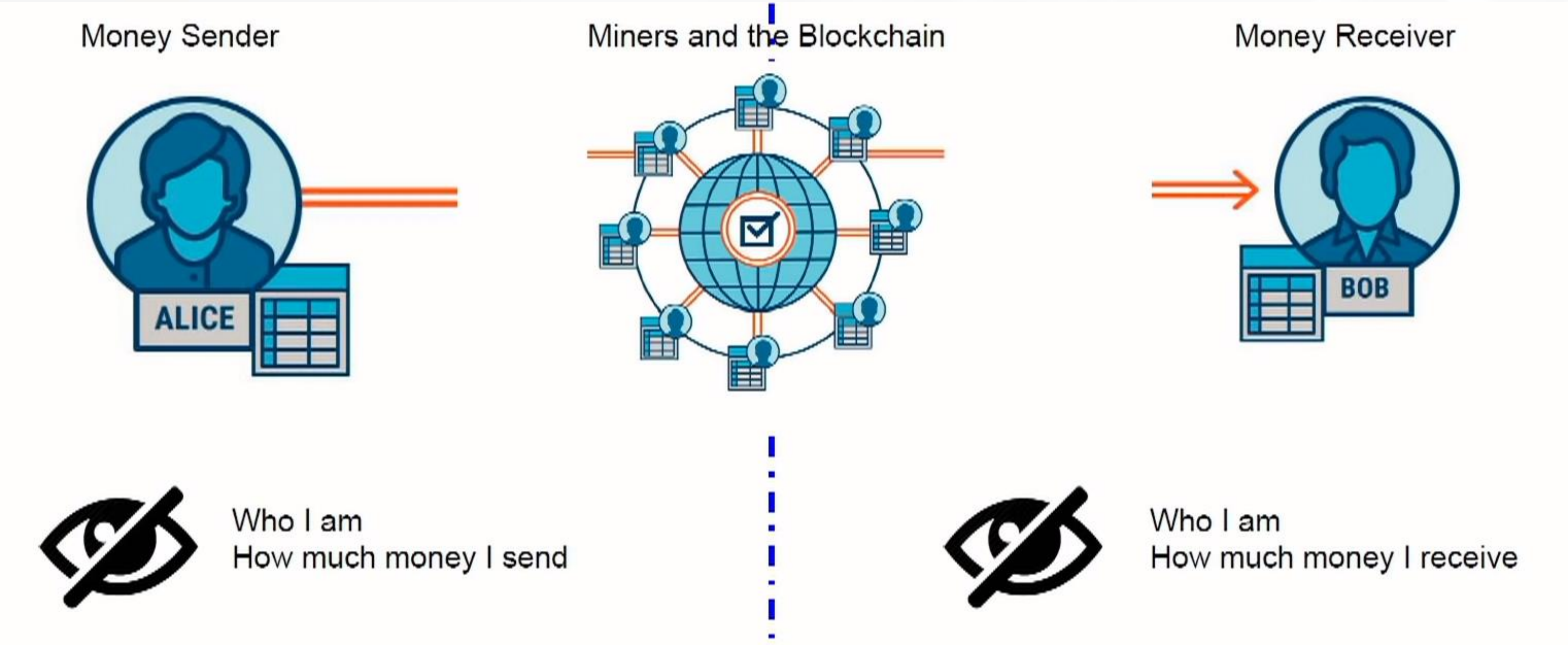


Transaction Details

Blockchain	Bitcoin ₿
Type	Transfer
Amount	94,504 **BTC** *($1,018,147,900 USD)*
Timestamp	2 weeks 6 days ago (Fri, 06 Sep 2019 03:30:05 UTC)
Hash	*4410c8d14ff9f87ceeed1d65cb58e7c7b2422b2d7529afc675208ce2ce09ed7d* View transaction in blockchain.info
From	Unknown *Multiple Addresses*
To	Unknown *37XuVSEpWW4trkfmvWzegTHQt7BdktSKUs* View address in blockchain.info

Transactions are public

Confidentiality and Anonymity



1 **Confidentiality**

hiding the transferred **amounts**

2 **Sender Anonymity**

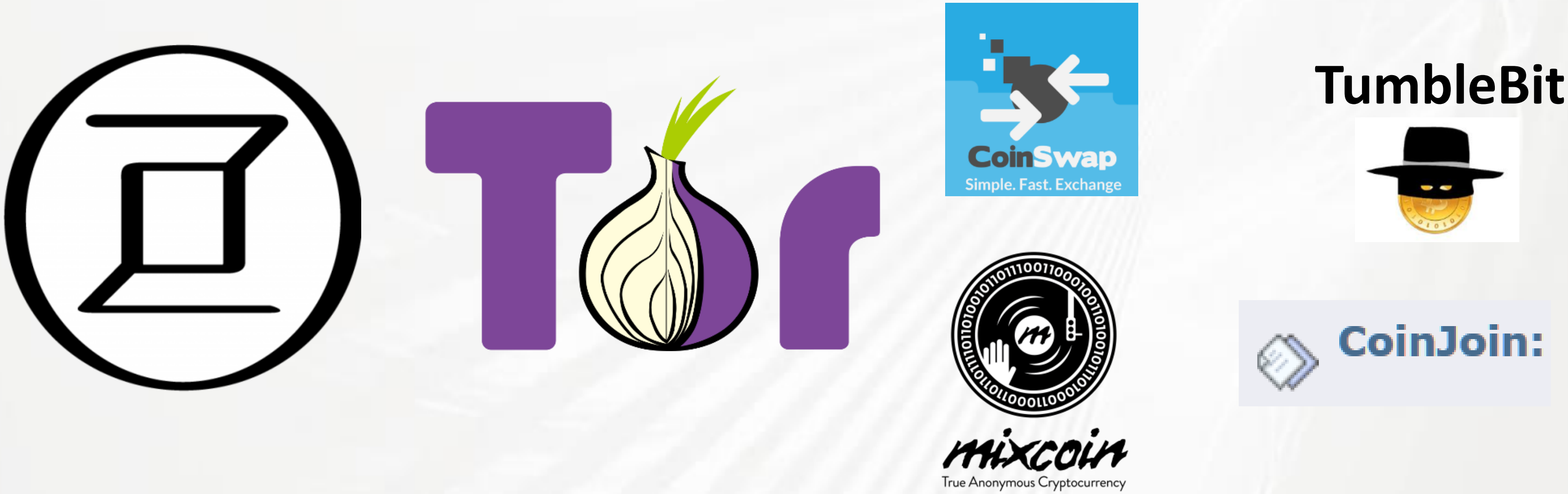
hiding the identities of the sender / the transaction origins

3 **Receiver Anonymity**

hiding the recipients identity

Bitcoin is not anonymous....what is next?

Option 1: minting/burning, mixers/tumblers compatible with Bitcoin



Option 2: New coin based on Zero Knowledge proofs

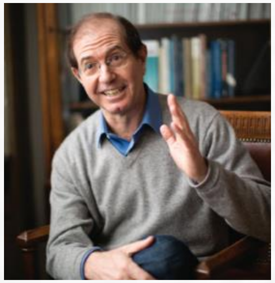
...

Zero-Knowledge Proofs

Sıfır Bilgi Ispatları

Introducing Zero-Knowledge Protocols [GMR'85]

Zero-Knowledge Proofs [Goldwasser-Micali-Rackoff'85]

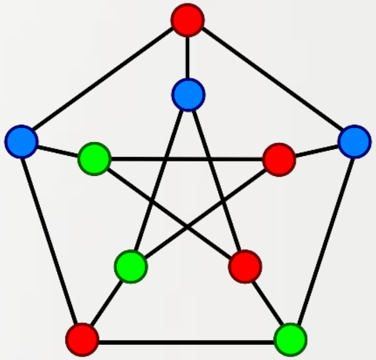


Every statement that has a classical proof (in **NP**) has zero-knowledge interactive proof, if **one-way functions** exists.

[Goldreich-Micali-Wigderson'91]

- There exists a ZK proof system for the NP-complete graph colouring problem with three colours.

[1] Goldreich, Oded; Micali, Silvio; Wigderson, Avi (1991). "Proofs that yield nothing but their validity". *Journal of the ACM*. **38** (3): 690–728.



http://web.mit.edu/~ezyang/Public/graph/svg.html

Secure Computation vs. Communication

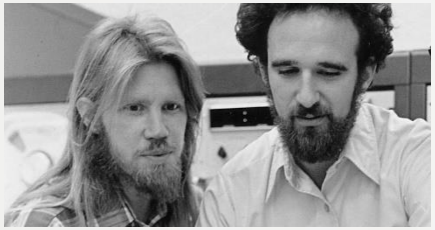
□ Secure Communication

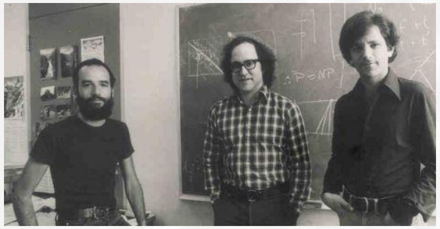
- Symetric-Key Cryptography
  - Block Ciphers
  - Stream Ciphers
  - Hash Functions
- Public-Key Cryptography
  - Asymmetric Encryption
  - Signature Schemes
- Access Control
- Etc.

≠

□ Secure Computation

- Secure Multi-party Computation
- Zero-Knowledge Protocols
  - Fiat-Shamir Protocol
  - Schnorr Proofs
  - Zk-Snarks
  - Zk-Starks
  - Bulletproofs
  - Sigma Bulletproofs etc.
- Private Function Evaluation
- Homomorphic Schemes
- Etc.





Can we achieve full privacy with ZKPs ?

ZKPs ≠ privacy

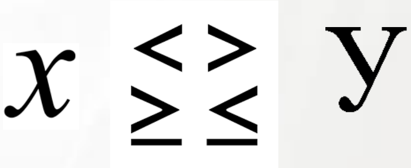
ZKPs == honest computation

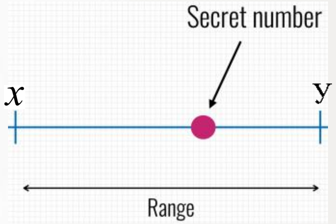
$$f(x) = y$$

+ proof

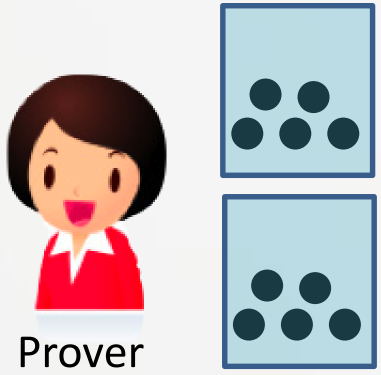
What you can prove using zero-knowledge proofs?

- There are four common statement types, though the following is not an exhaustive list:
  - • An **equality** statement (the subject’s bank account balance is equal to x), or **non-equality** statement.
  - • An **inequality** statement (the subject’s bank account balance exceeds x).
  - • A **range** statement (the subject’s bank account balance is within interval [a,b]), or out-of-range statement.
  - • A **membership** statement (the subject is on the client list of bank X), or non-membership statement.





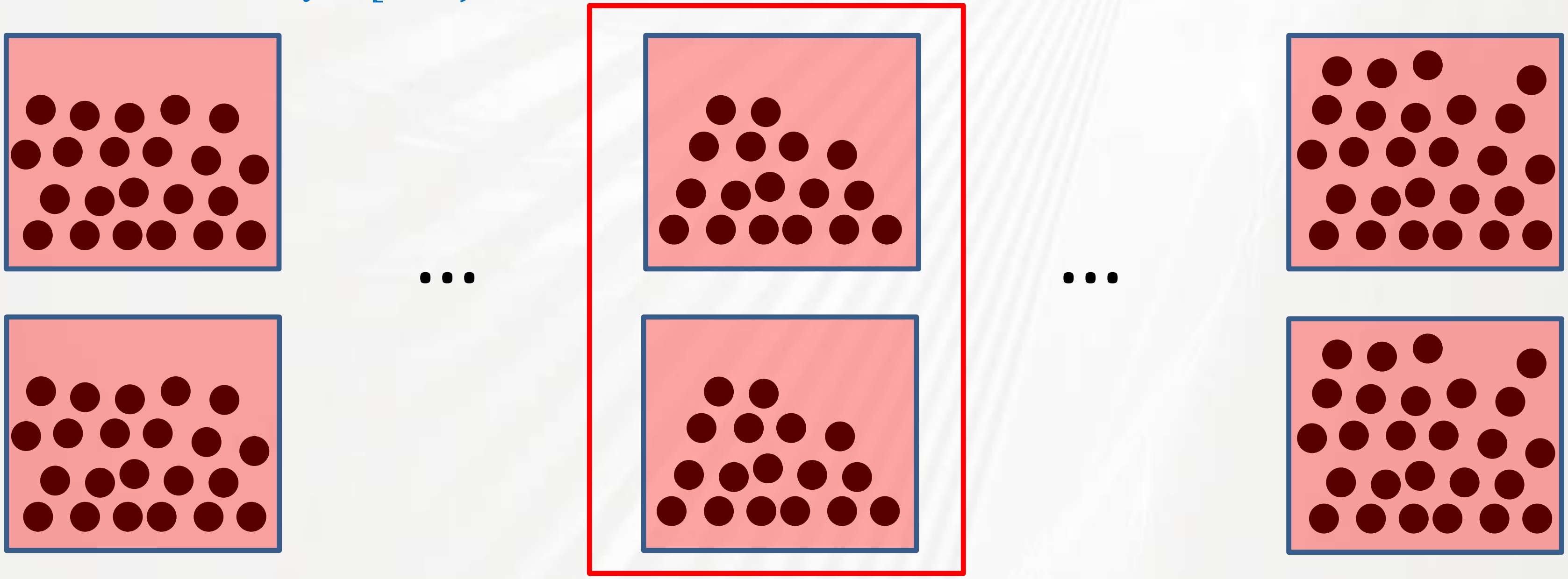
Zero-Knowledge Protocols – Equality Proof Example



Alice has two cups each containing $x \in [0, n)$ marbles.

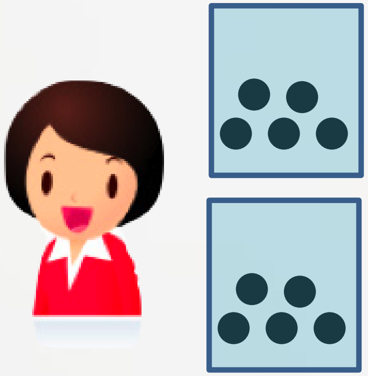
She wants to prove to Bob that both contain the same number without revealing x.

Alice prepares 10 pairs of buckets, both buckets in the i^{th} pair containing a random number $R_i \in [0, N)$ of marbles.



Bob chooses one of the pairs at random, and inspects the other 9 pairs to ensure that each pair indeed contains an identical number of marbles.

Zero-Knowledge Protocols – Equality Proof Example



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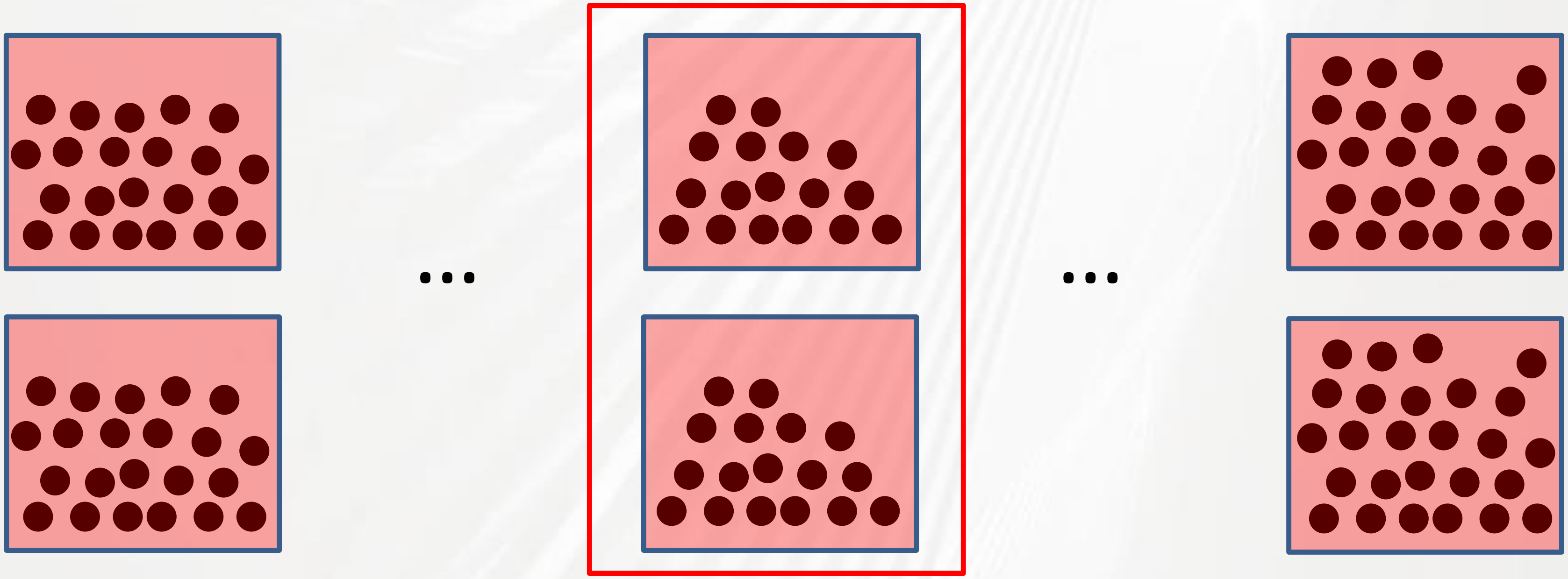
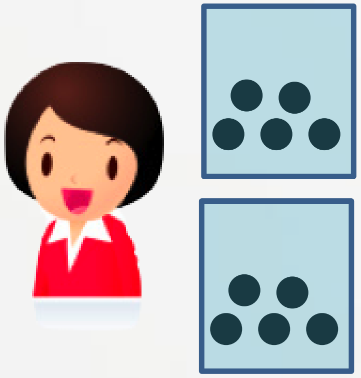


Zero-Knowledge Protocols – Example



Alice has two cups each containing $x \in [0, n)$ marbles.

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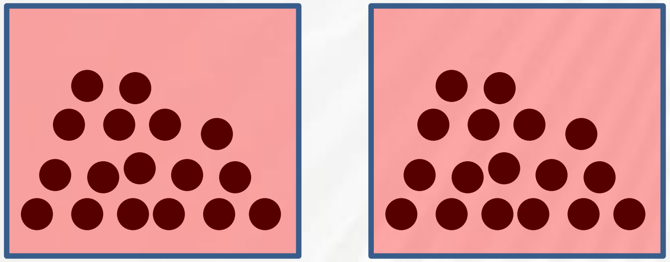
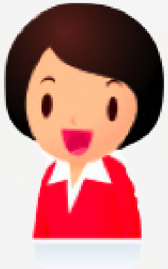
Alice pours the marbles from the first cup to the first bucket, and from the second cup to the second bucket.



Both contain $R_i \in_r [0, N)$ marbles

Zero-Knowledge Protocols – Equality Proof Example



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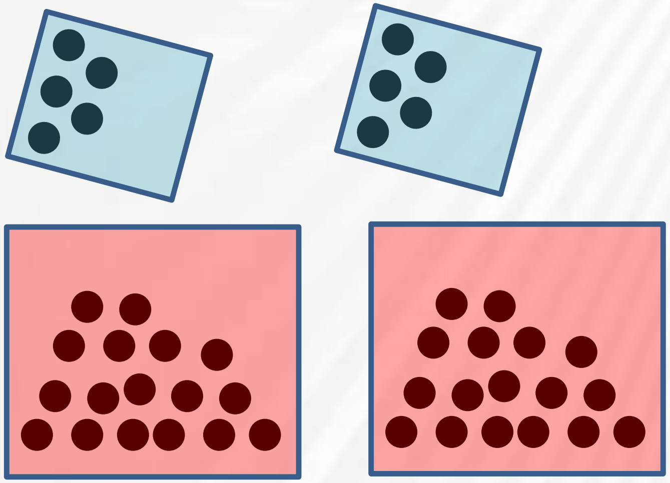

Both contain $R_i \in_r [0, N)$ marbles

Zero-Knowledge Protocols – Equality Proof Example

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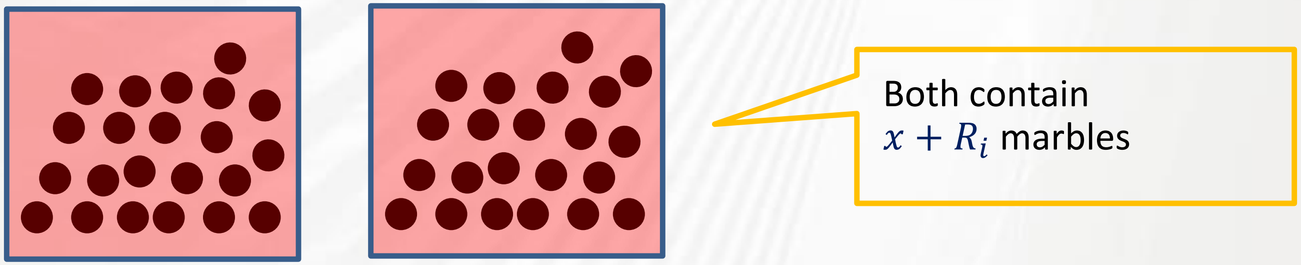


Both contain $x + R_i$ marbles

Bob accepts the proof if both buckets contain the same number of marbles.

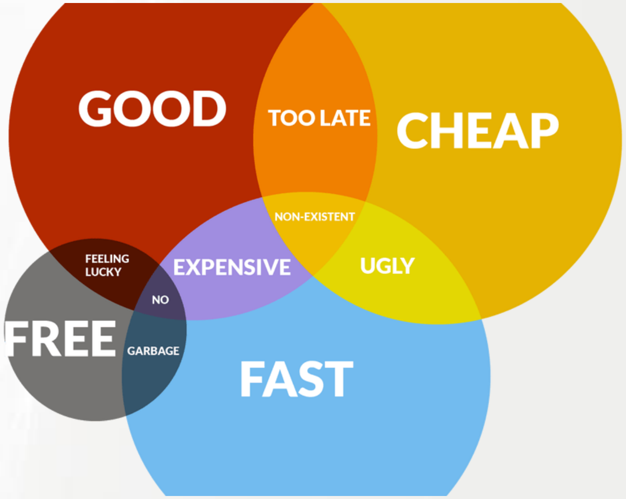
Soundness: If the cups contain a different number of marbles, Bob rejects with prob ≥ 0.9

Zero Knowledge: The number $x + R_i$ Bob sees is distributed n/N close to the uniform distribution on $(0, N]$. (Other 9 numbers are independent of X)

Credit to Boaz Barak ©

What is the success probability?

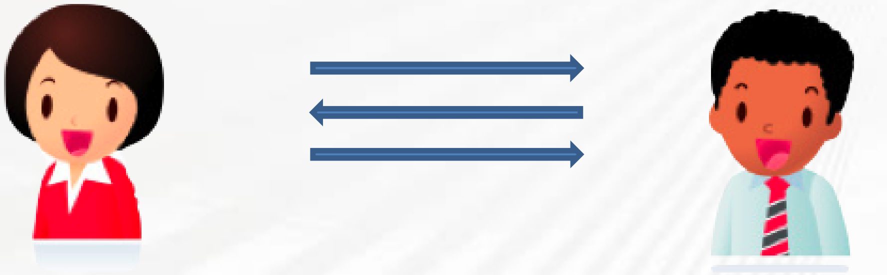
99,99..9 %



Properties of ZKP

□ Completeness:

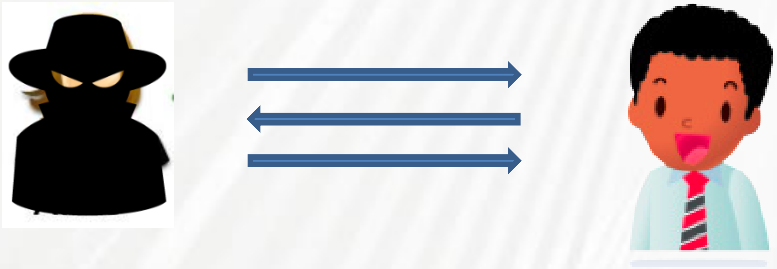
- if the statement is true, the honest verifier will be convinced of this fact by an honest prover.



→ accept

□ Soundness:

- if the statement is false, no cheating prover can convince the honest verifier that it is true, except with some small probability.

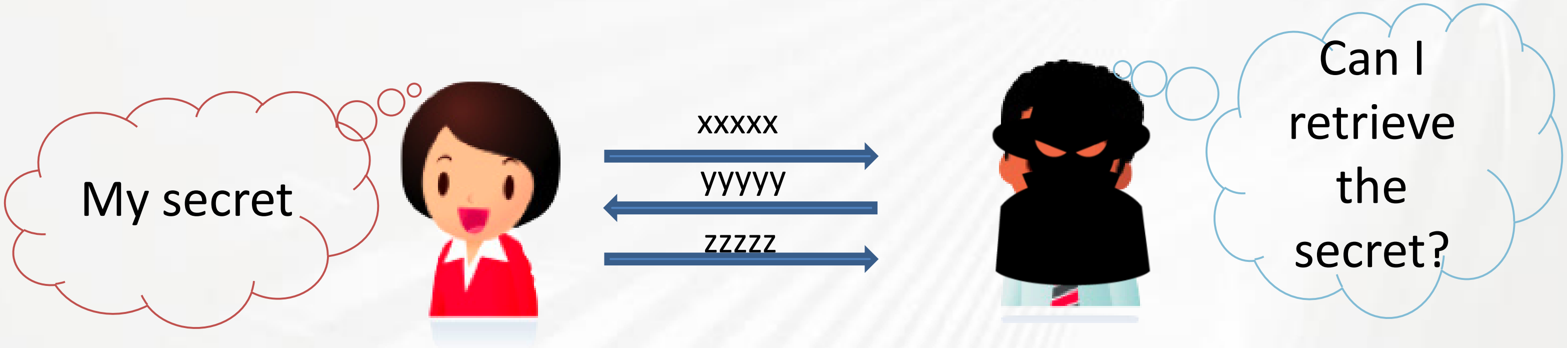


→ false

Properties of ZKP

❑ **Zero-knowledge**:

- if the statement is true, no verifier learns anything other than the fact that the statement is true.



My secret

xxxxx

yyyyy

zzzzz

Can I retrieve the secret?

Formalized by showing that every verifier has some *simulator* that, given only the statement to be proved (and no access to the prover), can produce a transcript that "looks like" an interaction between the honest prover and the verifier in question.

No secret

simulator

xxxxx

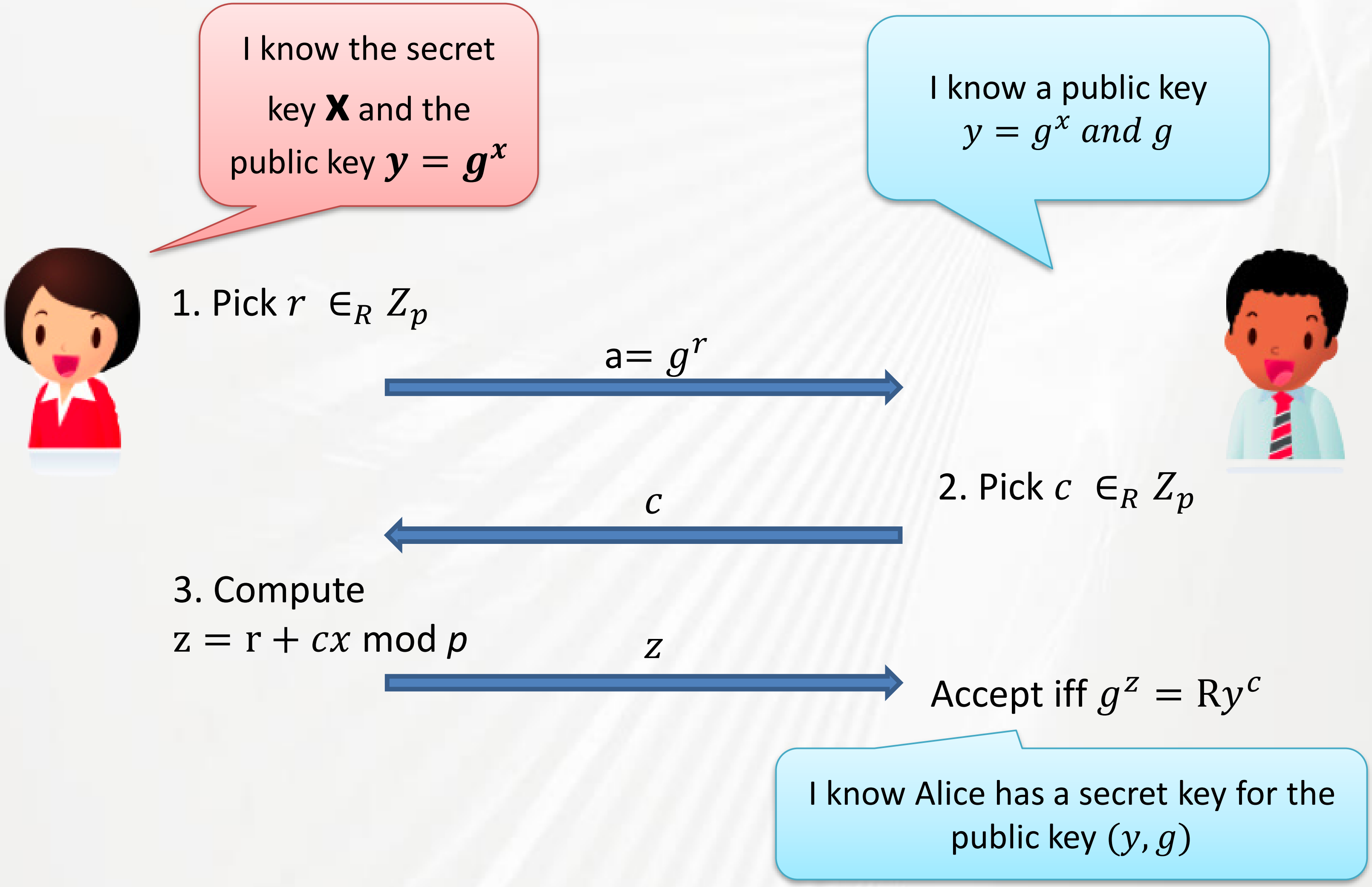
yyyyy

zzzzz

Zero-Knowledge Proof Schemes

Classical Schnorr Proofs	• C P Schnorr [1989] Efficient identification and signatures for smart cards, Crypto '89
zk-SNARKS	• E Ben-Sasson, A Chiesa, E Tromer, M Virza [2014] Succinct Non-Interactive Zero Knowledge for a von Neumann Architecture. USENIX'14
zk-STARKS	• E Ben-Sasson, I Bentov, Y Horesh, M Riabzev [2018] Scalable, transparent, and post-quantum secure computational integrity. e-print 2018/046
Bulletproofs	• B Bünz, J Bootle, D Boneh et al [2018] : Bulletproofs: Short Proofs for Confidential Transactions and More IEEE S&P'18.

Simple ZK proof - Schnorr’s Protocol



I know the secret key $\boldsymbol{x}$ and the public key $\boldsymbol{y = g^x}$

I know a public key $y = g^x\ and\ g$

1. Pick $r\ \in_R Z_p$

a= g^r

2. Pick $c\ \in_R Z_p$

c

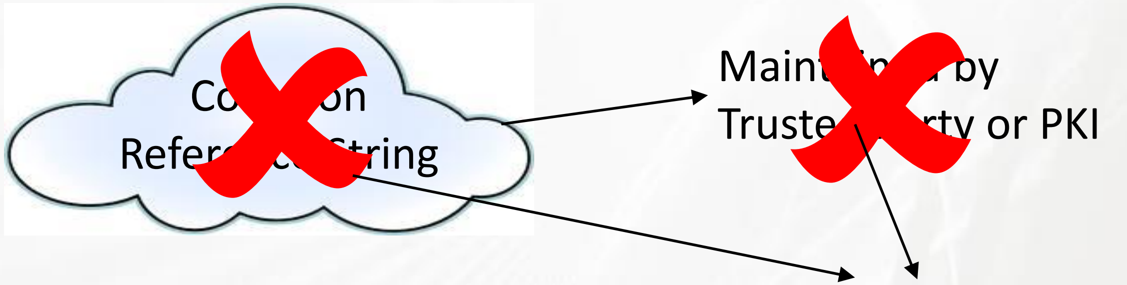
3. Compute $\mathrm{z} = \mathrm{r} + cx \bmod p$

z

Accept iff $g^z = \mathrm{R}y^c$

I know Alice has a secret key for the public key (y, g)

Variant: Non-Interactive ZK (NIZK)

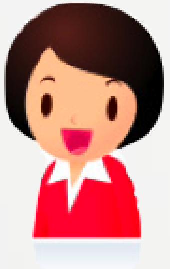


~~Common Reference String~~

~~Maintained by Trusted Party or PKI~~

Using Blockchains

I know the secret key **X** and the public key $\boldsymbol{y = g^x}$



1. Pick $r \in_R Z_p$, compute $R = g^r$

2. Pick $c = Hash(R, y, g)$

3. Compute $\mathrm{z} = \mathrm{r} + cx$ mod p

I know a public key $y = g^x$ *and* g

c, z

Compute $\mathrm{R} = g^z y^{-c}$

Accept iff $c = Hash(R, y, g)$

I know Alice has a secret key for the public key (y, g)

Confidential transaction in Monero

- The **amount confidentiality** is provided by using **Pedersen commitment**
  - The correctness **(= balance**) of the input and output amount is guaranteed by the **additive homomorphic** property of using Pedersen commitment.
  - But we still need to ensure that for every transaction amount M:

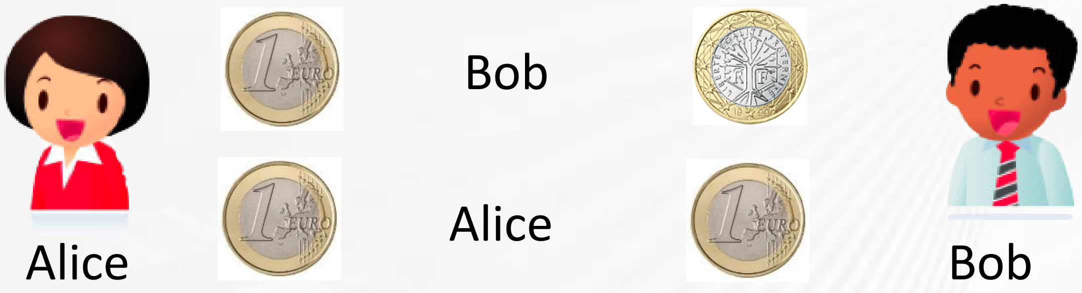
$$0 \le M < max$$

→ We need a (compact) **zero-knowledge range proof** for all transaction amount M!

⇩

- They use inner product argument (Bulletproof)
  - Represent each amount M as a binary vector $(a_1, a_2, \ldots, a_n)$
  - showed in ZK that M = $(a_1, a_2, \ldots, a_n)$ • $(1, 2, 4, 8, \ldots, 2^{k-1})$
  - → $0 \le M < 2^k$

Commitment Schemes



Alice

Bob

Alice

Bob

- Example:
  - Alice and Bob must agree who will clean tonight
  - They are at their offices. Each tosses a coin & they call:
    - If tosses are the same, then Alice cleans
    - If tosses are different, then Bob cleans
  - Who talks first?

Commitment Schemes

Alice

Alice

Bob

Bob

- ➢ Alice and Bob toss
  - Alice talks first

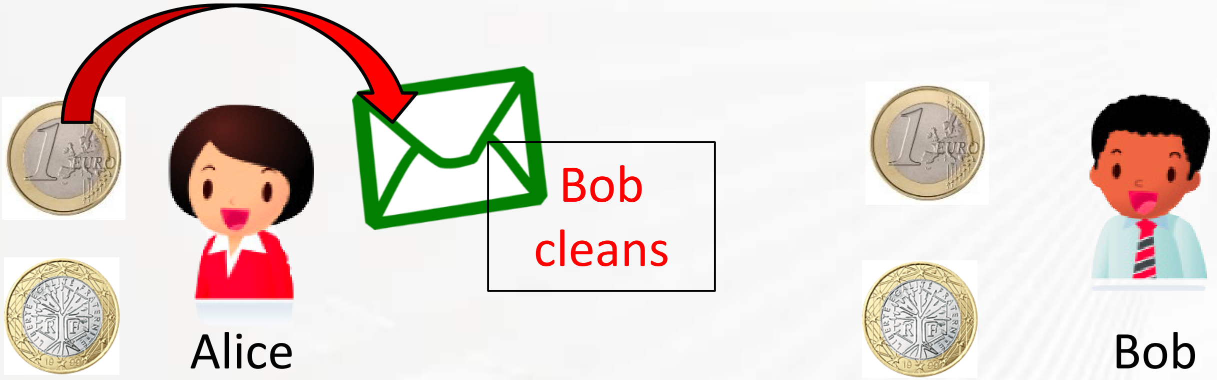
    Bob says he tossed the same value

  - Bob talks first

    Alice says she tossed the opposite value

- ➢ How can we avoid this?

Commitment Schemes



- Commitment: an envelope with a strange seal
  - Alice talks first
  - Commit phase: she hides toss in envelope, gives it to Bob
  - Bob reveals toss
  - Reveal phase: Alice tells Bob how to unseal envelope

Commitment Schemes

➢ Properties:

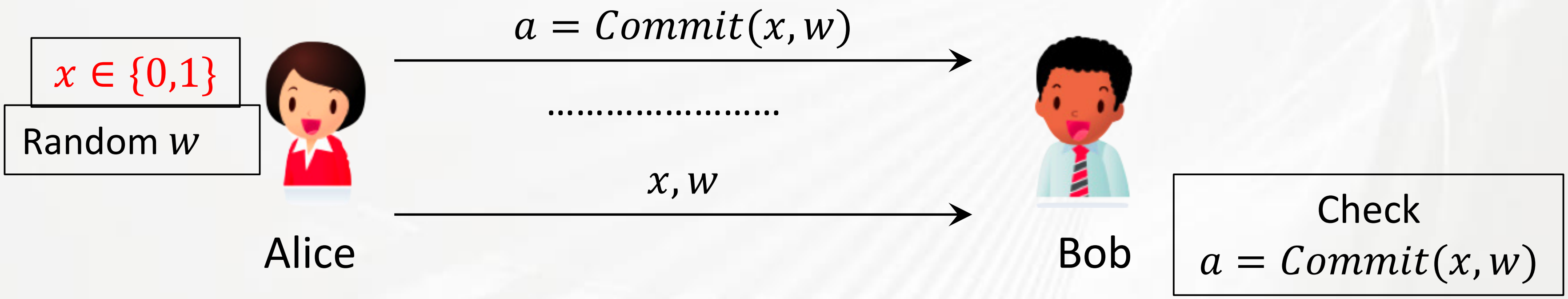
- Hiding: The content of the envelope is not visible

  Bob doesn't know anything about Alice's toss

- Binding: Alice can't change the content in the envelope

  Alice can't cheat after getting Bob's toss

Pedersen Commitments



$x \in \{0,1\}$

Random w

Alice

$a = Commit(x,w)$

……………………

x, w

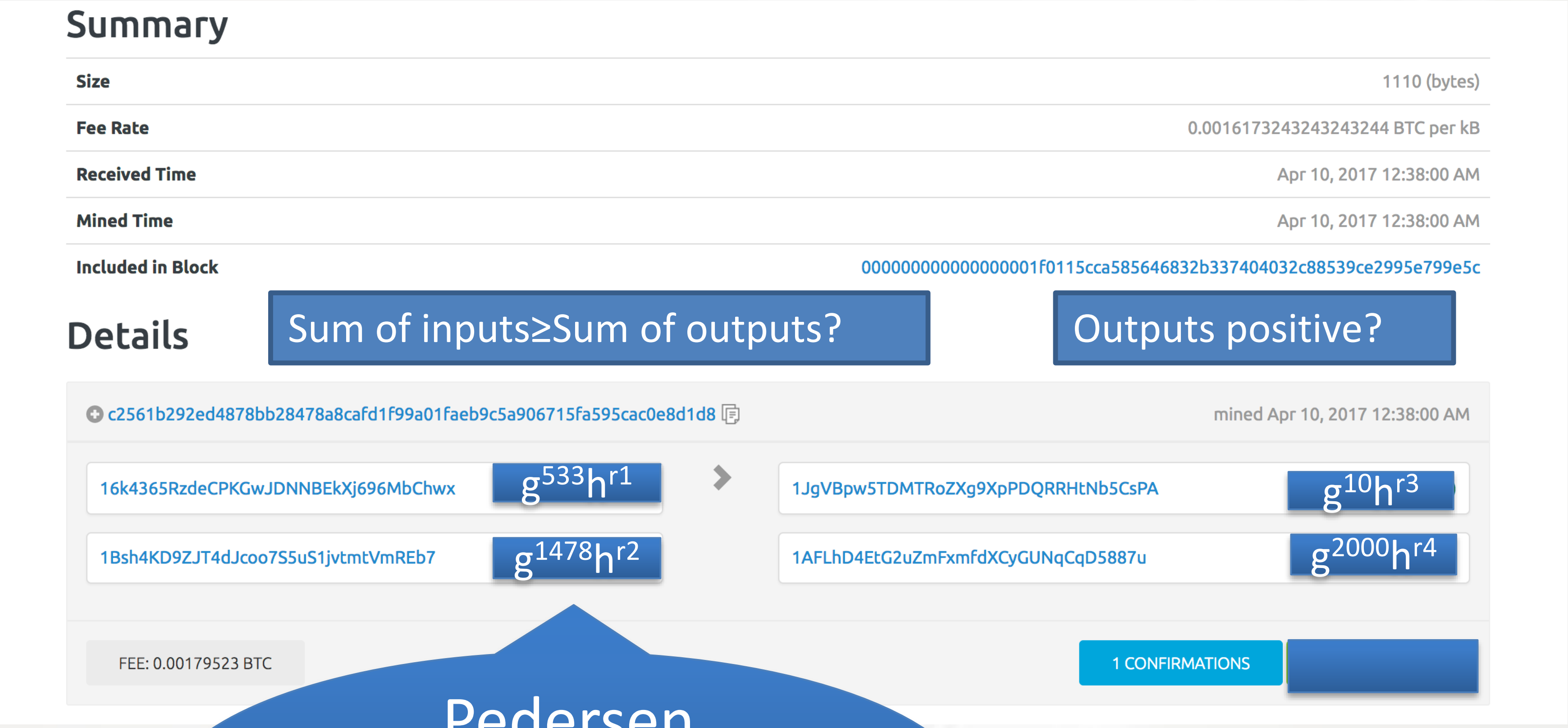
Bob

Check

$a = Commit(x,w)$

- Setup: $G_p^* = < g >$, prime field, $h = g^s \in G_p^* \setminus \{1\}$, s unknown
- Commitment of input value $x \in \{0,1\}$:
  - Choose random witness $w \leftarrow_R \{1, \ldots, p-1\}$
  - Compute $Commit(x,w) = g^w h^x = g^w g^{xs} = g^{w+xs}$
- Binding: Alice can’t change the content in the envelope? **Computational**
- Hiding: The content of the envelope is not visible ? **Info. Theoretical**

Confidential Transactions



Summary

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Received Time	Apr 10, 2017 12:38:00 AM
Mined Time	Apr 10, 2017 12:38:00 AM
Included in Block	0000000000000000001f0115cca585646832b337404032c88539ce2995e799e5c

Details

Sum of inputs≥Sum of outputs?

Outputs positive?

c2561b292ed4878bb28478a8cafd1f99a01faeb9c5a906715fa595cac0e8d1d8

mined Apr 10, 2017 12:38:00 AM

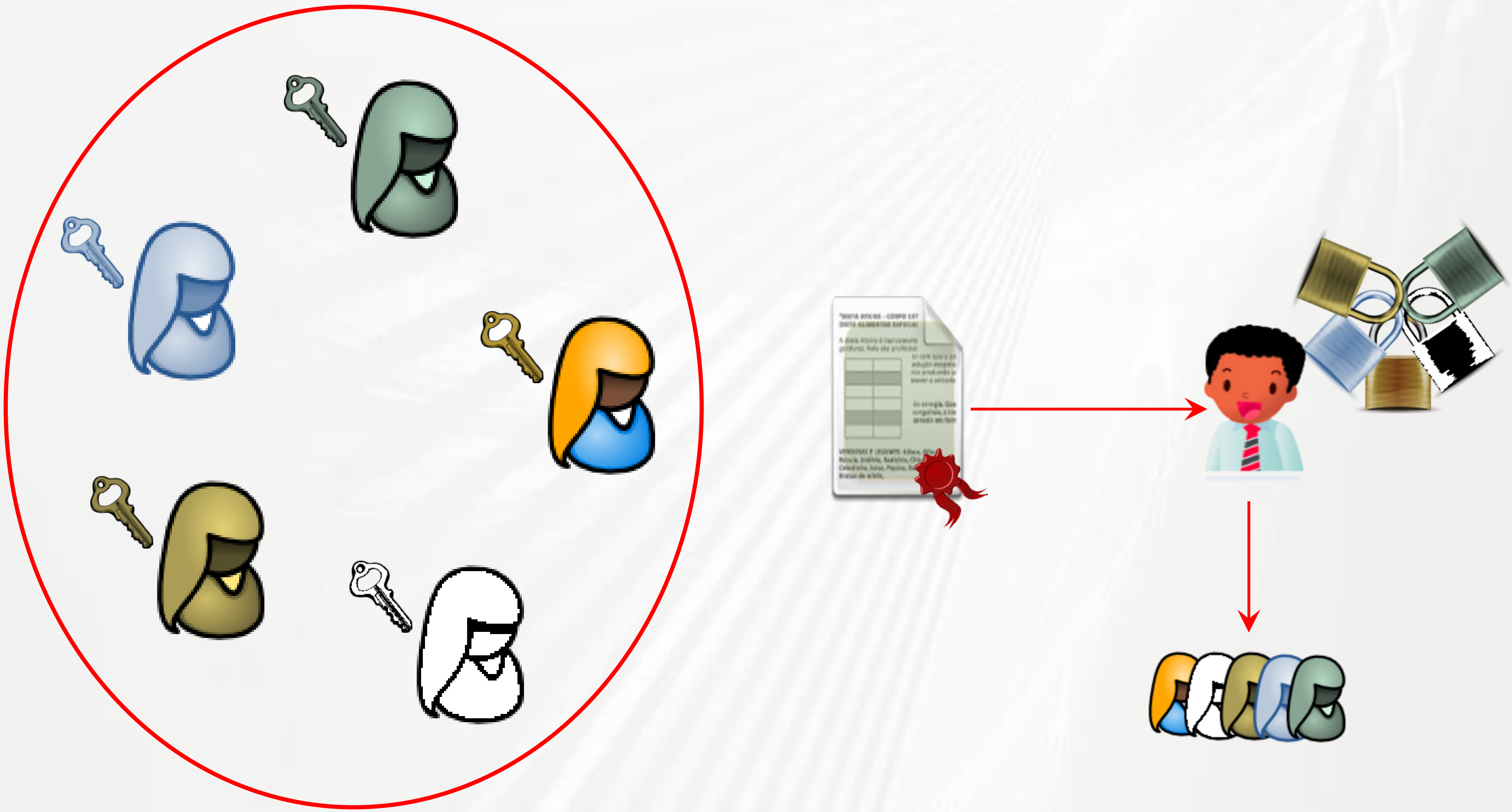
16k4365RzdeCPKGwJDNNBEkXj696MbChwx	$g^{533}h^{r1}$	1JgVBpw5TDMTRoZXg9XpPDQRRHtNb5CsPA	$g^{10}h^{r3}$
1Bsh4KD9ZJT4dJcoo7S5uS1jvtmtVmREb7	$g^{1478}h^{r2}$	1AFLhD4EtG2uZmFxmfdXCyGUNqCqD5887u	$g^{2000}h^{r4}$

FEE: 0.00179523 BTC

1 CONFIRMATIONS

Pedersen commitment: Commit(x;r)=g^xh^r

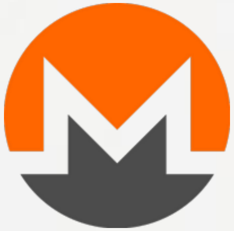
Ring Signatures



Credit to Cristina Onete ©

Projects using ZKPs

Bulletproofs



Use Bulletproofs for more efficient **range proofs only** and **not for privacy directly**

Bulletproofs (Range Proofs)

Proving that a number is within a range

$$\mathbf{v} \in [0,2^n)$$

Zero Knowledge about the Inner Product of Two Vectors

Any number can be represented as inner product of two vectors.

$$\mathbf{5 = <[1, 0, 1] , [2^2, 2^1, 2^0]>}$$

5 equals inner product of 2 vectors [1, 0, 1] and $[2^2, 2^1, 2^0]$

Bulletproofs (Range Proofs)

This is also how binary works

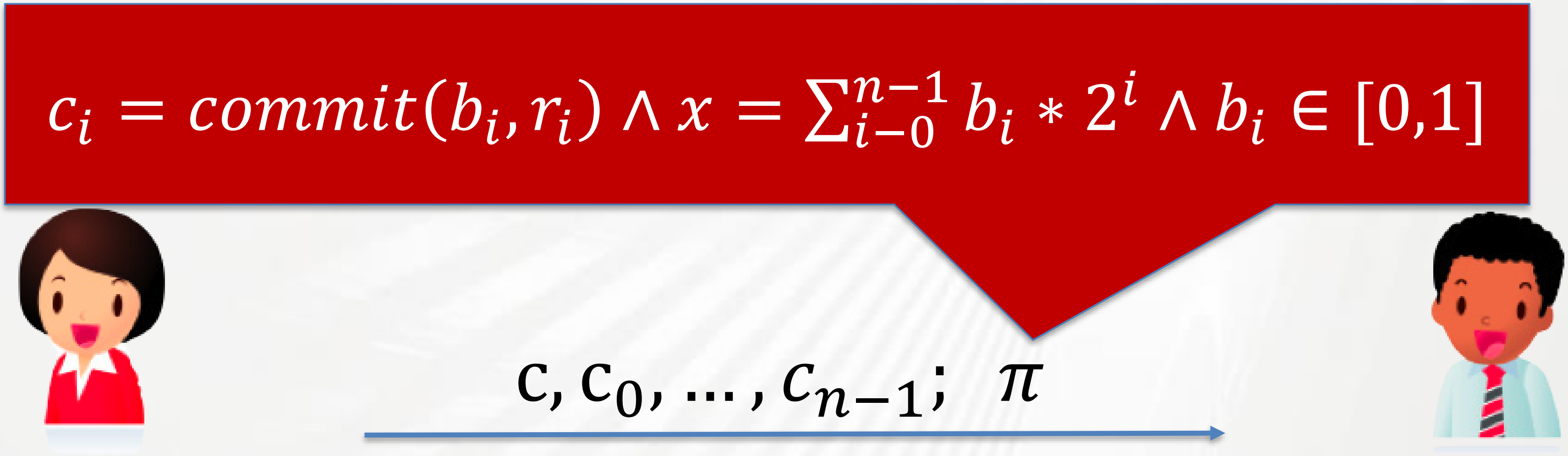
$101_{binary} = 5_{decimal}$ since $1(2^2) + 0(2^1) + 1(2^0)$

$v = \langle a, 2^n \rangle$

Example:
v = 5 and we wanted to prove that 5 is in range of 0 to 2^n **without showing 5**

$v \in [0,2^n)$

Concrete Range Proof using bit commitments



$$c_i = commit(b_i, r_i) \wedge x = \sum_{i-0}^{n-1} b_i * 2^i \wedge b_i \in [0,1]$$

$$\mathrm{c}, \mathrm{c}_0, \dots, c_{n-1}; \quad \pi$$

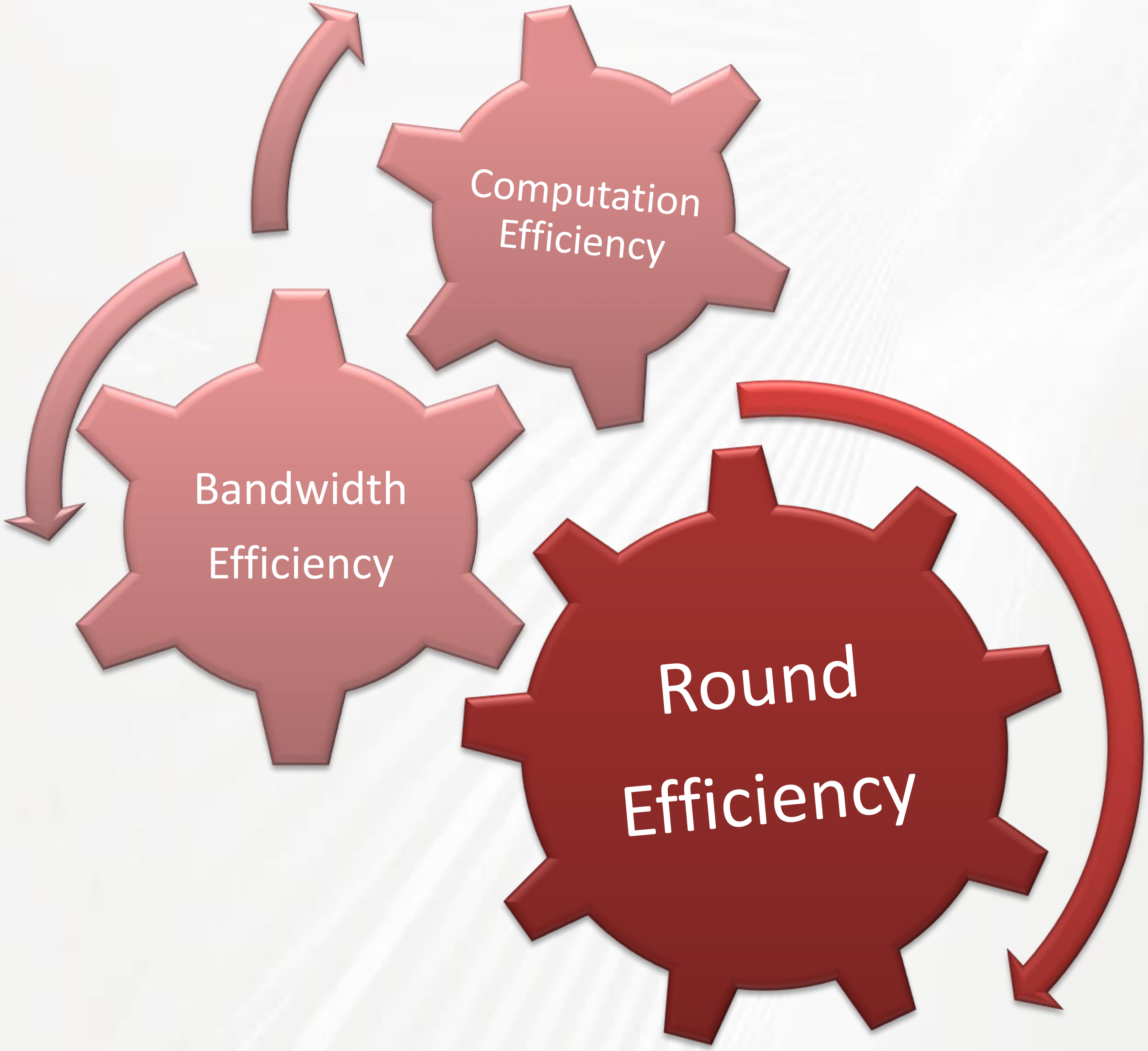
$$x = (b_0, \dots, b_{n-1}), b_i \in [0,1]$$

$$r_i \leftarrow \mathbb{Z}_q \forall\, i \in [0, n-1]$$

$$c_i = commit(b_i; r_i) \forall\, i \in [0, n-1]$$

Credit to Benedikt Bünz ©

Research Directions

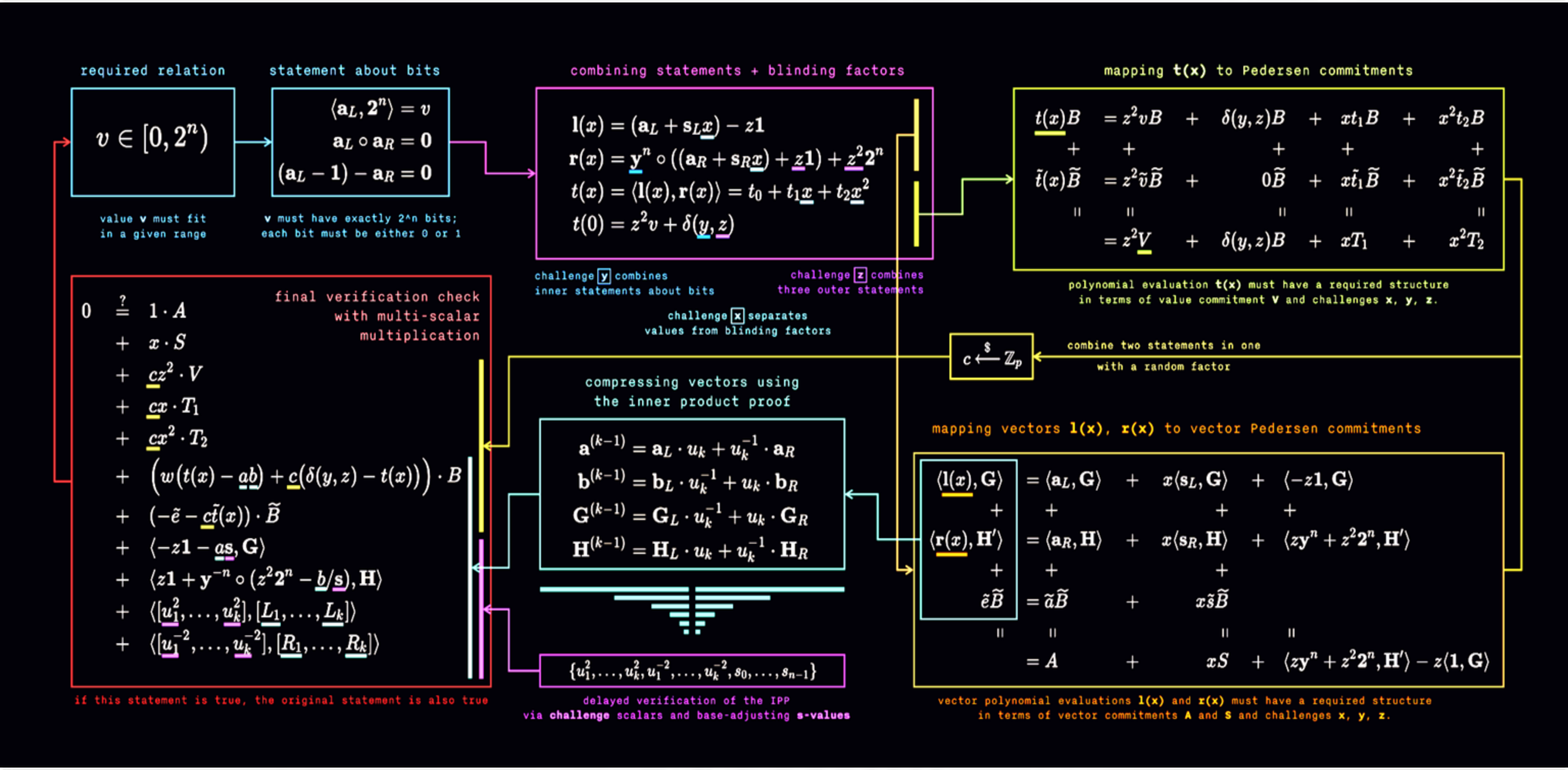


Computation Efficiency

Bandwidth Efficiency

Round Efficiency

Full Scheme of the Bullutproofs



required relation

$$v \in [0, 2^n)$$

value v must fit in a given range

statement about bits

$$\langle \mathbf{a}_L, \mathbf{2}^n \rangle = v$$
$$\mathbf{a}_L \circ \mathbf{a}_R = \mathbf{0}$$
$$(\mathbf{a}_L - \mathbf{1}) - \mathbf{a}_R = \mathbf{0}$$

v must have exactly 2^n bits; each bit must be either 0 or 1

combining statements + blinding factors

$$\mathbf{l}(x) = (\mathbf{a}_L + \mathbf{s}_L x) - z\mathbf{1}$$
$$\mathbf{r}(x) = \mathbf{y}^n \circ ((\mathbf{a}_R + \mathbf{s}_R x) + z\mathbf{1}) + z^2\mathbf{2}^n$$
$$t(x) = \langle \mathbf{l}(x), \mathbf{r}(x) \rangle = t_0 + t_1 x + t_2 x^2$$
$$t(0) = z^2 v + \delta(y, z)$$

challenge $\boxed{\mathbf{y}}$ combines inner statements about bits

challenge $\boxed{\mathbf{z}}$ combines three outer statements

challenge $\boxed{\mathbf{x}}$ separates values from blinding factors

mapping **t(x)** to Pedersen commitments

$$\begin{array}{cccccccccc}
t(x)B & = z^2 vB & + & \delta(y,z)B & + & xt_1B & + & x^2t_2B \\
+ & + & & + & & + & & + \\
\tilde{t}(x)\widetilde{B} & = z^2\tilde{v}\widetilde{B} & + & 0\widetilde{B} & + & x\tilde{t}_1\widetilde{B} & + & x^2\tilde{t}_2\widetilde{B} \\
\shortparallel & \shortparallel & & \shortparallel & & \shortparallel & & \shortparallel \\
 & = z^2 V & + & \delta(y,z)B & + & xT_1 & + & x^2T_2
\end{array}$$

polynomial evaluation **t(x)** must have a required structure in terms of value commitment **V** and challenges **x**, **y**, **z**.

$$c \xleftarrow{\$} \mathbb{Z}_p$$

combine two statements in one with a random factor

compressing vectors using the inner product proof

$$\mathbf{a}^{(k-1)} = \mathbf{a}_L \cdot u_k + u_k^{-1} \cdot \mathbf{a}_R$$
$$\mathbf{b}^{(k-1)} = \mathbf{b}_L \cdot u_k^{-1} + u_k \cdot \mathbf{b}_R$$
$$\mathbf{G}^{(k-1)} = \mathbf{G}_L \cdot u_k^{-1} + u_k \cdot \mathbf{G}_R$$
$$\mathbf{H}^{(k-1)} = \mathbf{H}_L \cdot u_k + u_k^{-1} \cdot \mathbf{H}_R$$

$$\{u_1^2, \dots, u_k^2, u_1^{-2}, \dots, u_k^{-2}, s_0, \dots, s_{n-1}\}$$

delayed verification of the IPP via **challenge** scalars and base-adjusting **s-values**

mapping vectors **l(x)**, **r(x)** to vector Pedersen commitments

$$\begin{array}{cccccccc}
\langle \mathbf{l}(x), \mathbf{G} \rangle & = \langle \mathbf{a}_L, \mathbf{G} \rangle & + & x\langle \mathbf{s}_L, \mathbf{G} \rangle & + & \langle -z\mathbf{1}, \mathbf{G} \rangle \\
+ & + & & + & & + \\
\langle \mathbf{r}(x), \mathbf{H}' \rangle & = \langle \mathbf{a}_R, \mathbf{H} \rangle & + & x\langle \mathbf{s}_R, \mathbf{H} \rangle & + & \langle z\mathbf{y}^n + z^2\mathbf{2}^n, \mathbf{H}' \rangle \\
+ & + & & + & & \\
\tilde{e}\widetilde{B} & = \tilde{a}\widetilde{B} & + & x\tilde{s}\widetilde{B} & & \\
\shortparallel & \shortparallel & & \shortparallel & & \shortparallel \\
 & = A & + & xS & + & \langle z\mathbf{y}^n + z^2\mathbf{2}^n, \mathbf{H}' \rangle - z\langle \mathbf{1}, \mathbf{G} \rangle
\end{array}$$

vector polynomial evaluations **l(x)** and **r(x)** must have a required structure in terms of vector commitments **A** and **S** and challenges **x**, **y**, **z**.

final verification check with multi-scalar multiplication

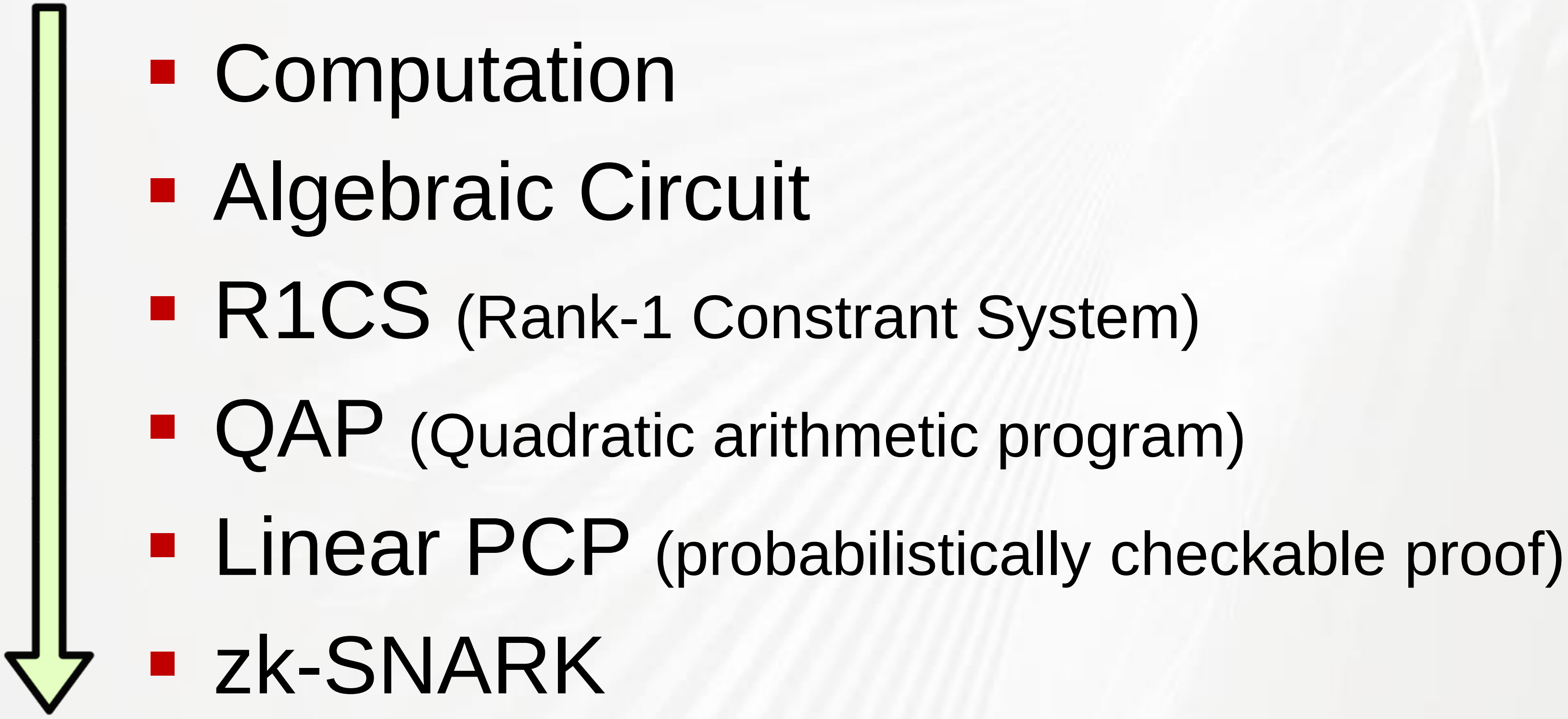
$$\begin{aligned}
0 \overset{?}{=}\ & 1 \cdot A \\
+\ & x \cdot S \\
+\ & cz^2 \cdot V \\
+\ & cx \cdot T_1 \\
+\ & cx^2 \cdot T_2 \\
+\ & \Big(w\big(t(x) - ab\big) + c\big(\delta(y,z) - t(x)\big)\Big) \cdot B \\
+\ & (-\tilde{e} - c\tilde{t}(x)) \cdot \widetilde{B} \\
+\ & \langle -z\mathbf{1} - a\mathbf{s}, \mathbf{G} \rangle \\
+\ & \langle z\mathbf{1} + \mathbf{y}^{-n} \circ (z^2\mathbf{2}^n - b/\mathbf{s}), \mathbf{H} \rangle \\
+\ & \langle [u_1^2, \dots, u_k^2], [L_1, \dots, L_k] \rangle \\
+\ & \langle [u_1^{-2}, \dots, u_k^{-2}], [R_1, \dots, R_k] \rangle
\end{aligned}$$

if this statement is true, the original statement is also true

https://github.com/dalek-cryptography/bulletproofs

B Bünz, J Bootle, D Boneh et al [2018] : Bulletproofs: Short Proofs for Confidential Transactions and More IEEE S&P’18.

zkSNARK construction via QAP and Linear PCPs



- Computation
- Algebraic Circuit
- R1CS (Rank-1 Constrant System)
- QAP (Quadratic arithmetic program)
- Linear PCP (probabilistically checkable proof)
- zk-SNARK

Credit to Eran Tromer ©

Properties of Zk-Snakrs

- Efficiency:
  - 288 byte **proof per transactions** (128-bit security)
  - <6 ms to **verify a proof**
  - <1 min to create

    for 2^{64} coins; asymptotically: log(#coins)
  - 896MB "system parameters"
    (fixed throughout system lifetime).

- **Trust in initial generation of system parameters (once).**

- Crypto assumptions:
  - Pairing-based elliptic-curve crypto
  - Less common: Knowledge of Exponent

    [Boneh Boyen 04] [Gennaro 04] [Groth 10] …
  - Properties of SHA256, encryption and signature schemes

Comparing Proof Systems (Oversimplified)

Proof System	Schnorr Σ-Protocol	Zk-SNARKs	STARKs	Bulletproofs
Proof Size	Long ✗	Very Short	Shortish	Shortish ✗
Prover	Linear	FFTs (memory req.)	FFT (Big memory req.) ✗	Multiexp.
Verifier	Linear ✗	Efficient	Efficient	Linear ✗
Trusted Setup	No	Required ✗	No	No
Practical	Yes	Yes	Not Quite ✗	Yes
Assumptions	Dlog + RO	Pairing +KoE	RO	Dlog + RO
Quantum Resistancy	No	No	Yes ✓	No

TÜBİTAK
BİLGEM
UEKAE

BLOKZİNCİR
Araştırma Laboratuvarı

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